Lorraine Sobson: Welcome to this Teaching Exceptional Children Podcast. I'm Lorraine Sobson, Publications Manager for the Council for Exceptional Children. Today, I'm talking with Elizabeth Hughes, Sarah Powell, and Liz Stevens, the authors of a recent article in Teaching Exceptional Children entitled “Supporting Clear and Concise Mathematics Language. Instead of That, Say This.”

Elizabeth Hughes is an assistant professor at Pennsylvania State University, Sarah Powell is an assistant professor at the University of Texas at Austin, and Liz Stevens is a doctoral student at UT Austin. Thank you all so much for joining me.

Elizabeth, you open the article with the idea that all children are mathematical language learners. Can you explain what this means?

Elizabeth Hughes: Thank you for having us. This is a great place to start. There’s this casual idea that mathematics is a universal language of numbers and symbols and, because the rules of math are constant and apply across cultures and dialect, math is a shared and universal language. As such, π is always 3.141592 and so forth and the area of a triangle is always one-half the length times the width, but this concept downplays the importance of words and terms we use to access and communicate this information.

I mean, just in my two examples, we have π, area, triangle, one-half length times and width. Students need to understand the language of math in order to access the content. Sometimes we don’t make this easy for students with exceptionalities: We often use mathematical language casually, but there are technical and exact definitions as they apply to math concepts.

As an example, in early childhood education, we may have children complete a connect-the-dots coloring page and give the directions to draw a line from one dot to the next. In math terms, a line is a straight path that extends in both directions without end. We're actually asking students to draw a line segment, which has two ending points.

It may sound trivial to say line segment instead of line, but as adults, we already have enough of an understanding of math and the flexibility of language that we may be able to use the term casually while understanding the technical mathematical meaning. As children are learning these terms and concepts for the first time, we need to be aware of what we are saying and how what we say may support or confuse students as they’re making sense of math, both now and in the future.
As special ed mathematics researchers, Sarah, Liz, and I are really interested in how language and vocabulary impacts students access to and understanding of mathematics. That's how this article came to be, through our conversation and research on the topic. We consider all children to be mathematical language learners because all children need to learn clear and concise math terms and their unique application in mathematical context.

Lorraine Sobson: Sarah, how does the mathematical language we use with younger students relate to their ability to understand more complex math in later grades?

Sarah Powell: That’s a really good question, Lorraine. The impetus behind the newer mathematics standards that we use here in the United States was to provide a consistency in mathematics across grade levels. This also needs to be true for the mathematical language that educators use with their students.

For younger students, when educators use more formal mathematics language and provide strong instruction about that language of mathematics, it will then be easier for students to understand mathematics in later grades.

For example, many elementary educators describe the inequality symbols—that’s the greater than symbol and the less than symbol—as an alligator that's hungry and wants to eat the number that's greater. Now, this might be cute, but later-grade teachers are not using the alligator for inequality—and, in fact, when describing inequalities with an alligator, that causes students to think there's only one inequality symbol that can be reversed when, in fact, these are two distinct symbols.

Now, in some of our research projects, we’ve taught students as young as first grade to use mathematical terms such as increase and decrease or balance and subtract. When taught well and practiced in meaningful ways, young children can learn and use the formal language of mathematics and this will help them not only in their grade level, but then it will help them across grade levels.

Lorraine Sobson: Liz, what kind of specific language should teachers use, for example, when introducing counting and cardinality?

Liz Stevens: Well, first it's important to define counting and cardinality. These are really just fancy terms for early numeracy concepts. In particular, cardinality refers to the understanding that the last number in a counting sequence represents the quantity in that set. For example, if a student counts a set of five toy cars, she knows that the last number said—in this case, 5— is the amount of cars in that set.

This domain is really important because it sets the foundation for everything else in mathematics. Students need a solid foundation in counting and cardinality, particularly for calculation skills later in their mathematical understanding.

As Sarah and Elizabeth have already mentioned, it’s important to use language that supports higher level thinking and integration of more complex skills later in
students' learning. Here's one way that we can use concise and accurate language within this…. Instead of saying, “1 is the first number,” when you're counting with students, say, "Let's start counting with 1." Or, "Let's start counting with 0." This is important because 1 is not the first number, [because] the number line extends infinitely in both directions.

Saying 1 is the first number could be problematic for students as they learn about 0 and negative integers. Even though kindergardeners won't be learning about negative integers, it's important to use math language that emphasizes we can start counting with many different numbers on a number line.

It's also helpful to pair counting instruction with a number line that extends in both directions. Again, this allows students to integrate more complex mathematical thinking and skills later in their schooling. This is just one example, but you can check out our article for more examples specific to counting and cardinality.

Lorraine Sobson: How can direct and accurate language help students understand place value? Elizabeth?

Elizabeth Hughes: Place value is such a core concept. It's more than just the location of a digit in a number. It really helps communicate the unique magnitude of the number on the number line, which Liz just referenced. Many errors that students make are as a result of not understanding a number magnitude and place value.

The way we talk about place value can reinforce the concepts. To discuss this one, let's think of an example when subtracting multi-digit numbers. 150 minus 18. When not considering the language of place value, we are at risk of explaining things that don't mathematically make sense.

To demonstrate this, I'm going to start off with a language non-example. When explaining how to solve the equation, 150 minus 18: “I can't take 8 from 0, so I need to borrow from the 5. I'll take one away from the 5, cross out the 5, put a 4, add my 1 to my 0 so I get 10. I can now do 10 minus 8 to get 2.

“Moving over to the next column, 4 minus 1 is 3, so I bring down my 3. 1 minus nothing is 1, so we can bring that down. My answer is 132.” The language I just used there is very procedural—and actually, at times, inaccurate, because we can't subtract 8 from 0. More importantly, it doesn't address what we are mathematically doing and why we are doing it.

Instead, if I use language to explain [how] to solve 150 minus 18 that acknowledges place value, it may sound more like this: “I know I have one group of 100s, five groups of 10, and zero groups of 1, from which I want to take away one group of 10 and eight 1s.
“Now, there are multiple ways that I can solve this problem. I may say that I can subtract 8 from 0, but that would give me a negative number and I don't really want to work with negative numbers right now. I want to work with positive numbers.

“Instead, I'm going to take one of my groups of 10 and regroup it as ten 1s. Now, 10 minus 8 is 2, which I put in the 1s place value of my answer. I have four groups of 10 left and I need to subtract one group of 10, which leaves three groups of 10. I can communicate I have three groups 10 ten by putting the digit 3 in my 10s place value of my answer.

“I have one group of 100s, from which I am taking away zero groups of 100s, leaving me with one group of 100. I can share this by putting the 1 digit in my 100s place value of my answer to give me 132.”

Now, in the second explanation, I more accurately explained what was happening and I did it in such a way that I could easily support the lesson with evidence-based practices such as using manipulatives. But unclear place value language extends beyond the use of just explaining how to solve equations; we are inundated with casual place value language.

To share that, here's another example. If I say, “5.04,” I'm sure our listeners know exactly what number I mean, but I didn't exactly say a number. I said the order of the digits in the number with the exception of O, which isn't a number at all. Yet most people could probably visualize what I was saying, but when we're working with students, instead of saying 5.04, I should say, "Five and four hundredths."

Again, it's important for us as adults to remember that we already have an understanding of place value, but students are just learning to make sense of place value so we can't take shortcuts with our math language, especially for students with exceptionalities that experience challenges processing components of language.

Lorraine Sobson: Sarah, how can language help with such things as number comparisons, operator symbols, and the equal sign?

Sarah Powell: Well, Lorraine, I already talked a little bit about number comparisons with the example of the inequality symbols and the alligator. By calling the inequality symbol the greater-than symbol and the less-than symbol, that would help students read inequalities in an easier manner. For example, 5 is greater than 2.

Now, as far as the operator symbols, educators need to be mindful that students will carry their mathematics language into subsequent grades. For example, let's talk about a minus sign. The minus sign can indicate that students are going to separate a quantity from a set.

For example, 9 minus 5. Maybe Elizabeth has nine apples and gives away five apples. We can ask about how many apples does Elizabeth have now? 9 minus 5.
Now, the minus sign can also indicate: compare two sets. For 9 minus 5, maybe Liz read nine books and I, Sarah, read five books. Then, you could ask, how many more books did Liz read than Sarah, or how many fewer books did Sarah read than Liz? It's still the same problem: 9 minus 5.

Now, imagine an educator taught students that the minus sign means “to take away” and even called the minus sign the takeaway sign or the takeaway symbol. That works really well for the story about Elizabeth and her apples, but it doesn't work at all for the story about Liz and Sarah and the books.

Now, when it comes to the equal sign, educators may say that the equal sign means “to write an answer,” and that is an incorrect understanding of the equal sign. In fact, the equal sign means that we need to balance two sides of an equation and educators should use that language. That is, they should describe the equal sign as meaning the same as. For example, 2 plus 5 is the same as 7.

Now, Lorraine, I could talk about the equal sign all day, but let's go ahead and go to the next question.

Lorraine Sobson: Your article points out the difficulty understanding rational numbers can be an indicator of later math difficulties. Can you explain why this is?

Sarah Powell: Yeah, longitudinal research, including that conducted by David Geary and other colleagues, indicates that an understanding of fraction concepts ... predicts later mathematics performance. That means that students with a more novice understanding of fractions exhibit mathematics difficulties in later grades.

As described by other researchers, knowledge about fractions is a gate keeper to upper-level mathematics, mainly because so much of mathematics after the introduction of rational numbers, including fractions, involves a strong understanding of rational numbers.

Lorraine Sobson: So, Elizabeth, what language should teachers use and avoid when explaining rational numbers?

Elizabeth Hughes: As Sarah talked about with rational numbers, we're referring to fractions, decimals, and percentages. These can be especially challenging because they do not follow the same rules as whole numbers, which children are taught first. For example, if you multiply two whole numbers, your product is of greater magnitude than the two numbers you multiplied. 2 times 2 is 4.

This is not necessarily true for rational numbers, where 2 times 1/2 is 1. In general, I would say that teachers should use language that express the unique magnitude of the fraction, decimal or percent and connect this to the number line.

This lends itself to the language we should avoid, like top number and bottom number, when talking about a fraction. Here's why: A fraction itself is a number with
a unique location on the number line. A fraction consists of a numerator, a divisor line, and a denominator, but these are all components communicating one number.

When we start talking about the top number and the bottom number, we run the risk of students conceptualizing the numerator and the denominator as two separate whole numbers. In turn, students may try to apply whole-number properties to fractions. We see this when students make errors adding fractions by adding across the numerator and adding across the denominator.

Another term we should avoid when working with fractions is reduce. Now, I grew up learning to reduce fractions but what I learned wasn't really reducing fractions. When we reduce, we make something smaller. We aren't really reducing $\frac{4}{6}$ to $\frac{2}{3}$, we're simplifying the fraction or finding an equivalent fraction in least terms.

Simplifying the fraction does not change the magnitude of the number, where reducing would. We can, however, reduce in geometry when we make a shape smaller.

Lorraine Sobson: How can imprecise language affect learning in geometry? For example, why is the word same a problem in geometry instruction? Liz?

Liz Stevens: This is a really good question. Students with mathematic difficulties struggle with geometry concepts through high school. Often, students are asked to describe geometric figures, two- and three-dimensional, and it's necessary that they are familiar with the formal geometric vocabulary to do so.

Students are held accountable for such understanding on high-stakes assessments, so it's important for us, as teachers, to use and model the language that students will be exposed to on those tests—but, most importantly, concise language during geometry instruction supports conceptualize understanding.

You asked about the word same. Sometimes, students use the word same when describing two shapes that have the same size and the same shape. Using same, however, is really just too vague and this can be problematic, particularly for students with math difficulty.

Students need to be able to distinguish between shapes that are similar, meaning they have the same shape, and those that are congruent, meaning they have the same size and the same shape. As students initially learn to describe shapes, it may be okay to use same, but eventually teachers and students should use the formal terms congruent and similar.

Another example is the term side. It's important to use this term only when describing two-dimensional shapes. A triangle has three sides and three angles. Students may use the term side when describing a 3-D shape too, but there aren't sides to a 3-D shape.
Instead of using *side*, use the terms *faces* and *edges*. A cube, for example, has six faces, and the place where two faces meet is called an *edge*. Making the distinction between the 2-D and 3-D terminology helps students to understand the difference between these figures.

Lorraine Sobson: Sarah, how do your language recommendations relate to existing math interventions that teachers might be using?

Sarah Powell: Lorraine, lots of educators are using formal mathematical language within their interventions. We observed that a lot of the time—but for those who are not, it would be relatively easy to introduce formal language alongside any current informal language. All we need to do is scaffold the language for students and then, with appropriate instruction, students can learn the terms that we use within intervention.

Within intervention, educators must be mindful and plan accordingly with language. That is, educators need to review the upcoming unit and understand the mathematical language that will be used. Then, educators need to provide concise and precise definitions for mathematical terms and use those definitions consistently within intervention.

If educators have to make up definitions about mathematical language on the fly, it can get you into trouble very quickly. It's important for educators not to only realize that they have to plan for teaching mathematical content, but they also have to plan for the mathematical language.

Elizabeth Hughes: Sarah makes some great points. To add to this, I think one of the strengths about what we're talking about today is that it be applied across situations. We can use it when implementing evidenced-based interventions but we can also use it in informal math conversations. We can start with this clear concise math language early.

Sometimes, I hear that young children aren't ready for technical math language, but children are going to learn what we effectively teach them. It may seem easier to simplify or informalize the language to a child's level, but this doesn't benefit our students—especially our students with exceptionalities—in the long run.

For students with exceptionalities [who] experience challenges processing language, we are requiring more of them to learn the informal language and then later learn the formal language. As educators, we can teach children to use technical terms and, as Sarah pointed out, scaffold our language to support their understanding of that term.

For example, I have two preschoolers at home. I try to use clear and concise math language when talking with them. I will scaffold the language to make it accessible on their language, but I always make sure that I scaffold it up to the correct term and not down to toddler language.
For example, we do what we call Vitamin Math in the morning. This is essentially where I use their vitamins as math manipulatives and we solve math problems before they eat their vitamins.

Between my two children, they eat four vitamins, so I take out four vitamins and I may ask, "What is the ratio of red vitamins to orange vitamins? Ratio means to compare values. In this case, we are comparing how many red vitamins we have, to how many orange vitamins we have. What is the ratio of red vitamins to orange vitamins?"

Through this, I may need to scaffold the process and ask follow-up questions, but I still bring it back to that word and have them use the word ratio back to me. Then later, I'll support that vocabulary development by using it consistently, having them use it consistently, and starting to generalize new situations.

My children use the math words because I have explicitly taught them the math words. As Sarah said, using clear and concise language requires that we not only plan for content, but that we purposely plan for language.

Lorraine Sobson: Vitamin Math. Elizabeth, I think you have given a lot of parents different ideas now about ways to get their kids to eat food using concise math language.

Liz, is there anywhere else that teachers can look for further guidance on this issue?

Liz Stevens: Definitely check out our article in TEC. We've talked about a few examples today, but you can find about 40 examples of ways teachers can formalize mathematics language in our article. We provide figures for each domain that outline what's commonly said and why that may be problematic as well as the alternative language we recommend and why that language supports students' mathematical understanding.

Two other resources you might want to check out are articles written by Karp, Bush, and Dougherty. The first talks about 13 rules that expire, which was published in *Teaching Children Mathematics*. The second article talks about rules that expire in the middle grades. That article was published in *Mathematics Teaching in the Middle School*.

These articles encourage teachers to think about the way they teach mathematics and how certain rules or tricks may make the task seem easy for students at first, but these rules can be problematic for students later in their math learning because they no longer apply, particularly as they encounter more complex mathematical tasks.

For example, telling students that when you add two numbers you get a bigger number, is a rule that expires. This may help students at first when they're learning addition, but can actually cause confusion later on when they start with working with negative integers. Check out those resources to think about how you teach mathematics as well as the language that you use.
Finally, we're embarking on a middle-school version of this article next. If any listeners out there have suggestions related to language used when teaching mathematics to middle school students, please reach out to us. We'd love to hear your suggestions.

Lorraine Sobson: Thanks so much for joining me today.

Sarah Powell: You're welcome.

Elizabeth Hughes: Thank you for having us.

Liz Stevens: Thank you so much.

Lorraine Sobson: The article we've been discussing, “Supporting Clear and Concise Mathematics Language: Instead of That, Say This,” appears in Volume 49 of *TEACHING Exceptional Children*. *TEACHING Exceptional Children* is a publication of the Council for Exceptional Children. To learn more about CEC, visit cec.sped.org.